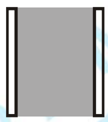
FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Tuesday 07th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

1. A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1 + \alpha x)$ where 'x' is the distance measured from one of the plates. If $(\alpha d) << 1$, the total capacitance of the system is best given by the expression :



(1) $\frac{AK\varepsilon_0}{d}\left(1+\frac{\alpha d}{2}\right)$ (2) $\frac{A\varepsilon_0 K}{d}\left(1+\left(\frac{\alpha d}{2}\right)^2\right)$

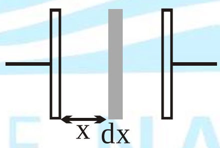
(3) $\frac{A\varepsilon_0 K}{d}\left(1+\frac{\alpha^2 d^2}{2}\right)$ (4) $\frac{AK\varepsilon_0}{d}(1+\alpha d)$

NTA Ans. (1)

Sol. As K is variable we take a plate element of Area A and thickness dx at distance x

Capacitance of element

$$dC = \frac{(A)K(1+\alpha x)\varepsilon_0}{dx}$$



Now all such elements are is series so equivalent capacitance

$$\frac{1}{C} = \int \frac{1}{dC} = \int_0^d \frac{dx}{AK\varepsilon_0(1+\alpha x)}$$

$$\frac{1}{C} = \frac{1}{\alpha AK\varepsilon_0}\ln\left(\frac{1+\alpha d}{1}\right)$$

$$= \frac{1}{C} = \frac{1}{\alpha AK\varepsilon_0}\left(\alpha d - \frac{(\alpha d)^2}{2} + \frac{(\alpha d)^3}{3} + ...\right)$$

$$\Rightarrow \frac{1}{C} = \frac{\alpha d}{\alpha AK\varepsilon_0}\left(1 - \frac{\alpha d}{2} + \frac{(\alpha d)^2}{3} + ...\right)$$

$$\frac{1}{C} = \frac{d}{AK\varepsilon_0}\left(1 - \frac{\alpha d}{2}\right)$$

$$C = \frac{AK\varepsilon_0}{d}\left(1 + \frac{\alpha d}{2}\right)$$

2. The time period of revolution of electron in its ground state orbit in a hydrogen atom is 1.6×10^{-16} s. The frequency of revolution of the electron in its first excited state (in s^{-1}) is:

(1) 6.2×10^{15} (2) 5.6×10^{12}

(3) 7.8×10^{14} (4) 1.6×10^{14}

NTA Ans. (3)

Sol. Time period of revolution of electron in n^{th} orbit

$$T = \frac{2\pi r}{V} = \frac{2\pi a_0\left(\frac{n^2}{Z}\right)}{V_0\left(\frac{Z}{n}\right)}$$

$$\Rightarrow T \propto \frac{n^3}{Z^2}$$

$$\frac{T_2}{T_1} = \frac{(2)^3}{(1)^3} = 8 \Rightarrow T_2 = 8 \times 1.6 \times 10^{-16}$$

Now frequency $f_2 = \frac{1}{T_2} = \frac{10^{16}}{8\times 1.6} \approx 7.8 \times 10^{14}$ Hz.

3. A long solenoid of radius R carries a time (t)-dependent current $I(t) = I_0 t(1 - t)$. A ring of radius 2R is placed coaxially near its middle. During the time interval $0 \le t \le 1$, the induced current (I_R) and the induced EMF(V_R) in the ring change as :

(1) At t = 0.5 direction of I_R reverses and V_R is zero

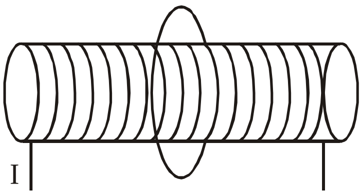
(2) Direction of I_R remains unchanged and V_R is zero at t = 0.25

(3) Direction of I_R remains unchanged and V_R is maximum at t = 0.5

(4) At t = 0.25 direction of I_R reverses and V_R is maximum

NTA Ans. (1)

Sol.



Magnetic flux (ϕ) through ring is $\phi = \pi(R)^2.B$

$\phi = (\pi R^2)(\mu_0 nI) = (\pi R^2 \mu_0 n I_0)(t - t^2)$

Induced e.m.f. of $V_R = \frac{-d\phi}{dt}$

$= \left(\pi R^2 \mu_0 n I_0\right)(2t-1)$

and induced current $I_R = \frac{\pi R^2 \mu_0 n I_0 (2t-1)}{R_R}$

($R_R \to$ Resistance of Ring)

Clearly V_R and I_R are zero at $t = \frac{1}{2}$ = 0.5 sec.

and their sign also changes at t = 0.5 sec.

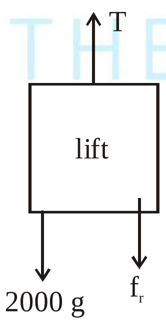
4. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to: (1 HP = 746 W, g = 10 ms^{-2})

(1) 1.7 ms^{-1} (2) 2.0 ms^{-1}

(3) 1.9 ms^{-1} (4) 1.5 ms^{-1}

NTA Ans. (3)

Sol.



Let elevator is moving upward with constant speed V.

Tension in cable

T = 2000 g + f_r = 2000 + 4000

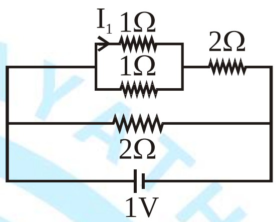
T = 24000 N

Power P = TV

$\Rightarrow$ 60 × 746 = (24000) V

$V = \frac{60 \times 746}{24000} = 1.865 \approx 1.9$ m/s.

5. The current I_1 (in A) flowing through 1 Ω resistor in the following circuit is :

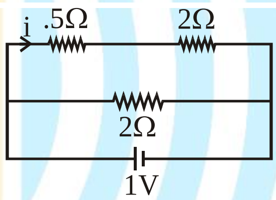


(1) 0.5 (2) 0.2

(3) 0.25 (4) 0.4

NTA Ans. (2)

Sol. Equivalent resistance of upper branch of circuit R = 2.5 Ω



Voltage across upper branch = 1 V

$\Rightarrow i = \frac{1}{2.5} = .4$ A

$\Rightarrow I_1 = 0.2$ A

6. A litre of dry air at STP expands adiabatically to a volume of 3 litres. If γ = 1.40, the work done by air is : ($3^{1.4} = 4.6555$) [Take air to be an ideal gas]

(1) 90.5 J (2) 48 J

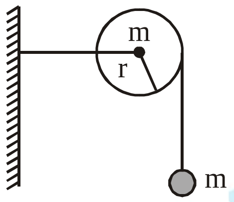
(3) 60.7 J (4) 100.8 J

NTA Ans. (1)

Sol. $w = \frac{nR(T_1 - T_2)}{\gamma - 1} = \frac{P_1V_1 - P_2V_2}{0.4}$

$$= \frac{100 - \frac{100}{4.6555} \times 3}{0.4} = 88.90 \cdot$$

7. As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be :



(1) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$ (2) $r\sqrt{\frac{3}{4gh}}$ (3) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$ (4) $r\sqrt{\frac{3}{2gh}}$

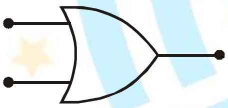
NTA Ans. (3)

Sol. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{1}{2}mr^2\times\frac{v^2}{r^2} = \frac{3}{4}mv^2$

$u = \sqrt{\frac{4}{3}gh}$

$\omega = \frac{v}{r}$

8. Which of the following gives a reversible operation?

(1)


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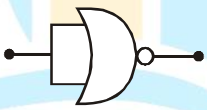

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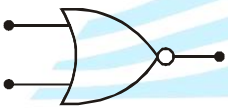
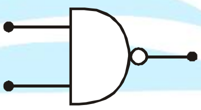


(4)


NTA Ans. (2)

9. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece, should be close to :

(1) 22 mm (2) 12 mm
(3) 33 mm (4) 2 mm

NTA Ans. (1)

Sol. $m = \frac{LD}{f_e \times f_0} = \frac{150\times 250}{f_e \times 25} = 375$

f_e = 20 mm.

10. The radius of gyration of a uniform rod of length l, about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it, is :

(1) $\frac{1}{8}l$ (2) $\sqrt{\frac{7}{48}}l$ (3) $\sqrt{\frac{3}{8}}l$ (4) $\frac{1}{4}l$

NTA Ans. (2)

Sol. $m\frac{l^2}{12} + m\frac{l^2}{16} = mk^2$

$\frac{7l^2}{48} = k^2$

11. If the magnetic field in a plane electromagnetic wave is given by $\vec{B} = 3\times10^{-8}\sin\left(1.6\times10^3x + 48\times10^{10}t\right)\hat{j}$T, then what will be expression for electric field?

(1) $\vec{E} = \left(9\sin\left(1.6\times10^3x + 48\times10^{10}t\right)\hat{k}\ V/m\right)$

(2) $\vec{E} = \left(3\times10^{-8}\sin\left(1.6\times10^3x + 48\times10^{10}t\right)\hat{i}\ V/m\right)$

(3) $\vec{E} = \left(60\sin\left(1.6\times10^3x + 48\times10^{10}t\right)\hat{k}\ V/m\right)$

(4) $\vec{E} = \left(3\times10^{-8}\sin\left(1.6\times10^3x + 48\times10^{10}t\right)\hat{j}\ V/m\right)$

NTA Ans. (1)

Sol. $\vec{E}\times\vec{B} = \vec{C} = -\hat{i}$

where $\vec{B}$ is along $\hat{j}$

$\frac{E}{B} = C$

E = 3 × 10^{-8} × 3 × 10^{8} = 9 V/m.

12. Consider a circular coil of wire carrying constant current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i. The magnetic flux through the area of the circular coil area is given by ϕ_0. Which of the following option is correct ?

(1) $\phi_i = -\phi_0$ (2) $\phi_i = \phi_0$
(3) $\phi_i < \phi_0$ (4) $\phi_i > \phi_0$

NTA Ans. (1)

13. Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm^2) is 90 ms^{-1}. If the Young's modulus of wire is 16×10^{11} Nm^{-2}, the extension of wire over its natural length is :

(1) 0.02 mm (2) 0.04 mm

(3) 0.03 mm (4) 0.01 mm

NTA Ans. (3)

Sol. $v = \sqrt{\frac{T}{\mu}}$

$$90 = \sqrt{\frac{\frac{YA}{l}\Delta l}{\frac{m}{l}}} = \sqrt{\frac{16 \times 10^{11} \times 10^{-6} \times \Delta l}{6 \times 10^{-3}}}$$

$$= \frac{8100 \times 3}{8} \times 10^{-8} = \Delta l$$

14. Visible light of wavelength 6000×10^{-8} cm falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction minimum is at 60° from the central maximum. If the first minimum is produced at θ_1, then θ_1 is close to :

(1) 20° (2) 45° (3) 30° (4) 25°

NTA Ans. (4)

Sol. $\sin\theta = \frac{2\lambda}{\omega}$

$$\sin 60° = \frac{2\lambda}{\omega}$$

$$\sin\theta_1 = \frac{\lambda}{\omega} = \frac{\sqrt{3}}{4}$$

$\theta_1 = 25°$

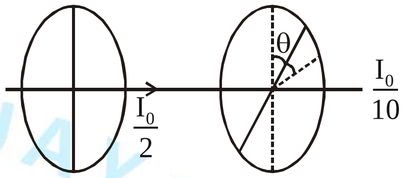
15. A polarizer - analyser set is adjusted such that the intensity of light coming out of the analyser is just 10% of the original intensity. Assuming that the polarizer - analyser set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero, is :

(1) 18.4° (2) 71.6° (3) 90° (4) 45°

NTA Ans. (1)

Sol. $\frac{I_0}{10} = I = \frac{I_0}{2} \times \cos^2\theta$

$$\cos\theta = \frac{1}{\sqrt{5}}$$



$\theta = 63.44°$

angle rotated = 90 – 63.44° = 26.56°

Closest is 1.

16. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth):

(1) $\frac{m}{20}\left(u - \sqrt{\frac{2GM}{3R}}\right)^2$

(2) $5m\left(u^2 - \frac{119}{200}\frac{GM}{R}\right)$

(3) $\frac{3m}{8}\left(u + \sqrt{\frac{5GM}{6R}}\right)^2$

(4) $\frac{m}{20}\left(u^2 + \frac{113}{200}\frac{GM}{R}\right)$

NTA Ans. (2)

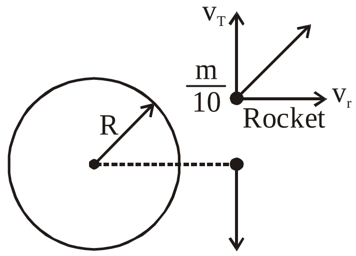
Sol. Applying energy conservation

$K_i + U_i = K_f + U_f$

$$\frac{1}{2}mu^2 + \left(-\frac{GMm}{R}\right) = \frac{1}{2}mv^2 - \frac{GMm}{2R}$$

$$v = \sqrt{u^2 - \frac{GM}{R}} \qquad ....(i)$$

By momentum conservation, we have



$$\frac{m}{10}v_T = \frac{9m}{10}\sqrt{\frac{GM}{2R}} \qquad ....(ii)$$

& $\frac{m}{10}v_r = mv$

$$\Rightarrow \frac{m}{10}v_r = m\sqrt{u^2 - \frac{GM}{R}} \qquad ....(iii)$$

Kinetic energy of rocket

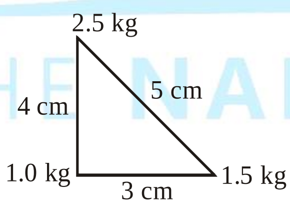
$$= \frac{1}{2}m\left(v_T^2 + v_r^2\right)$$

$$= \frac{m}{20}\left(81\frac{GM}{2R} + 100u^2 - 100\frac{GM}{R}\right)$$

$$= \frac{m}{20}\left(100u^2 - \frac{119GM}{2R}\right)$$

$$= 5m\left(u^2 - \frac{119GM}{200R}\right).$$

17. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point:



(1) 1.5 cm right and 1.2 cm above 1 kg mass

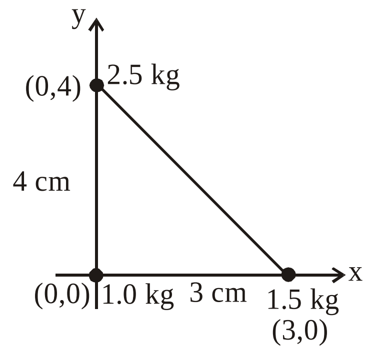
(2) 0.9 cm right and 2.0 cm above 1 kg mass

(3) 0.6 cm right and 2.0 cm above 1 kg mass

(4) 2.0 cm right and 0.9 cm above 1 kg mass

NTA Ans. (2)

Sol.



Let 1 kg as origin and x-y axis as shown

$$x_{cm} = \frac{1(0)+1.5(3)+2.5(0)}{5} = 0.9 \text{ cm}$$

$$y_{cm} = \frac{1(0)+1.5(0)+2.5(4)}{5} = 2 \text{ cm}$$

18. Two moles of an ideal gas with $\frac{C_P}{C_V} = \frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_P}{C_V} = \frac{4}{3}$. The value of $\frac{C_P}{C_V}$ for the mixture is:

(1) 1.50 (2) 1.42

(3) 1.45 (4) 1.47

NTA Ans. (2)

Sol. For 1st gas $\frac{C_{P_1}}{C_{V_1}} = \frac{5}{3} \Rightarrow C_{P_1} = 5x$ and $C_{V_1} = 3x$

For 2nd gas $\frac{C_{P_2}}{C_{V_2}} = \frac{4}{3} \Rightarrow C_{P_2} = 4x$ and $C_{V_2} = 3x$

Now for mixture $C_P = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 + n_2} = \frac{17R}{5}$

$$C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \frac{12R}{5}$$

$$\Rightarrow \frac{C_P}{C_V} = \frac{2(5x)+3(4x)}{2(3x)+3(3x)} = \frac{17}{12}$$

$$\Rightarrow \frac{C_P}{C_V} \approx 1.42.$$

19. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the correct equivalence would be:

(1) $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

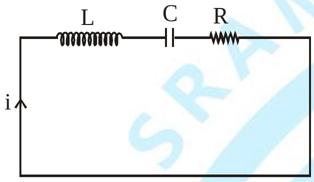
(2) $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

(3) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

(4) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

NTA Ans. (1)

Sol.



By kVL

$$-L\frac{di}{dt}-\frac{q}{C}-iR=0$$

$$L\frac{d^2q}{dt^2}+\frac{1}{C}q+R\frac{dq}{dt}=0$$

for damped oscillator

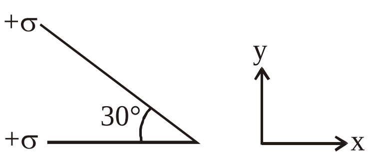
net force = $-kx - bv = ma$

$$\frac{md^2x}{dt^2}+kx+\frac{bdx}{dt}=0$$

by comparing ; Equivalence is

$$L \rightarrow m\ ;\ C \rightarrow \frac{1}{K}\ ;\ R \rightarrow b.$$

20. Two infinite planes each with uniform surface charge density + σ are kept in such a way that the angle between them is 30°. The electric field in the region shown between them is given by:



(1) $\frac{\sigma}{\varepsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y}+\frac{\hat{x}}{2}\right]$

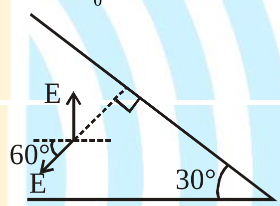
(2) $\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

(3) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}+\frac{\hat{x}}{2}\right]$

(4) $\frac{\sigma}{2\varepsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

NTA Ans. (2)

Sol. Electric field due to each sheet is uniform and equal to $E=\frac{\sigma}{2\varepsilon_0}$

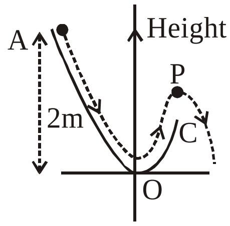


Now net electric field between plates

$$\vec{E}_{net}=E\cos 60°(-\hat{x})+(E-E\sin 60°)(\hat{y})$$

$$=\frac{\sigma}{2\varepsilon_0}\left[-\frac{\hat{x}}{2}+\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}\right].$$

21. A particle (m = 1 kg) slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaching its highest point P (height 1 m), the kinetic energy of the particle (in J) is : (Figure drawn is schematic and not to scale; take $g=10\ ms^{-2}$)_____.



NTA Ans. (10)

Sol. Mechanical energy conservation between A & P

$$U_1 + \cancelto{0}{K_1} = K_2 + U_2$$

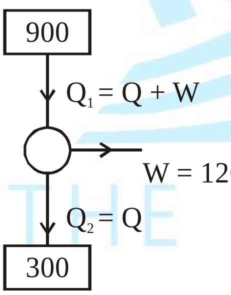
$$mg \times 2 = mg \times 1 + K_2$$

$$K_2 = mg \times 1 = 10 \text{ J.}$$

22. A Carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir, in a cycle, is____.

NTA Ans. (600)

Sol.



for carnot engine

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\frac{Q+1200}{Q} = \frac{900}{300}$$

$$Q + 1200 = 3Q$$

$$Q = 600 \text{ J.}$$

23. A beam of electromagnetic radiation of intensity 6.4×10^{-5} W/cm^2 is comprised of wavelength, λ = 310 nm. It falls normally on a metal (work function φ = 2eV) of surface area of 1 cm^2. If one in 10^3 photons ejects an electron, total number of electrons ejected in 1 s is 10^x. (hc=1240 eVnm, 1eV=1.6×10^{-19} J), then x is____.

NTA Ans. (10)

Sol. Power incident P = I × A

n = no. of photons incident/second

$$nE_{ph} = IA$$

$$n = \frac{IA}{E_{ph}}$$

$$n = \frac{IA}{\left(\frac{hc}{\lambda}\right)} = \frac{6.4 \times 10^{-5} \times 1}{\frac{1240}{310} \times 1.6 \times 10^{-19}}$$

$n = 10^{+14}$ per second

Since efficiency = 10^{-3}

no. of electrons emitted = 10^{+11} per second.

x = 11.

24. A non-isotropic solid metal cube has coefficients of linear expansion as :

5×10^{-5}/°C along the x-axis and 5×10^{-6}/°C along the y and the z-axis. If the coefficient of volume expansion of the solid is $C \times 10^{-16}$/°C then the value of C is ______.

NTA Ans. (60)

Sol. $\gamma = \alpha_x + \alpha_y + \alpha_z$

$= 5 \times 10^{-5} + 5 \times 10^{-6} + 5 \times 10^{-6}$

$= (50 + 5 + 5) \times 10^{-6}$

$\gamma = 60 \times 10^{-6}$

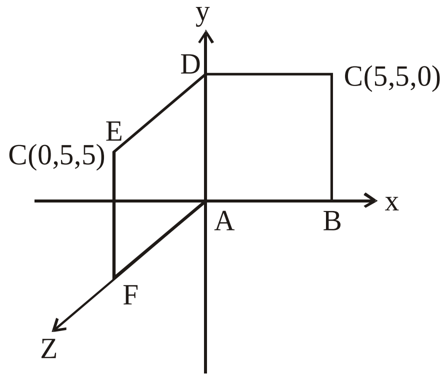
C = 60.

25. A loop ABCDEFA of straight edges has six corner points A(0,0,0), B(5,0,0), C(5,5,0), D(0, 5, 0), E(0, 5, 5) and F(0, 0, 5). The magnetic field in this region is $\vec{B} = (3\hat{i} + 4\hat{k})T$.

The quantity of flux through the loop ABCDEFA (in Wb) is _______ .

NTA Ans. (175)

Sol.



$\vec{A}_{ABCD} = 25\hat{k}$

$\vec{A}_{ADEF} = 25\hat{i}$

$\vec{A}_{net} = 25\hat{i} + 25\hat{k}$

$\vec{B} = 3\hat{i} + 4\hat{k}$

$\phi = \vec{B}.\vec{A}$

$= 25 \times 3 + 25 \times 4$

$\phi = 175\ W_{b.}$

FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Tuesday 07th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

CHEMISTRY

TEST PAPER WITH ANSWER & SOLUTION

1. A solution of m-chloroaniline, m-chlorophenol and m-chlorobenzoic acid in ethyl acetate was extracted initially with a saturated solution of $NaHCO_3$ to give fraction A. The left over organic phase was extracted with dilute NaOH solution to give fraction B. The final organic layer was labelled as fraction C. Fractions A, B and C, contain respectively :

(1) m-chlorobenzoic acid, m-chloroaniline and m-chlorophenol

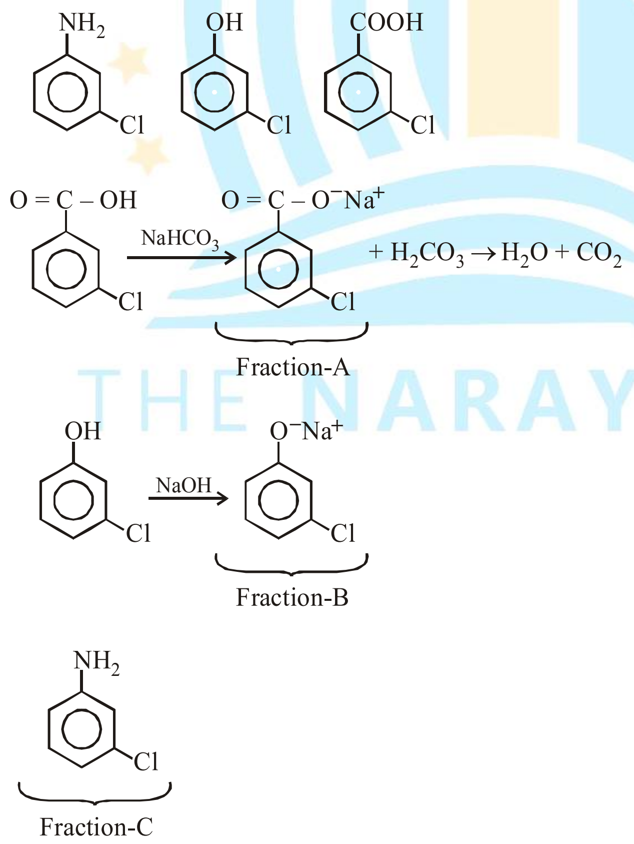
(2) m-chloroaniline, m-chlorobenzoic acid and m-chlorophenol

(3) m-chlorobenzoic acid, m-chlorophenol and m-chloroaniline

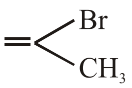
(4) m-chlorophenol, m-chlorobenzoic acid and m-chloroaniline

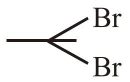
NTA Ans. (3)

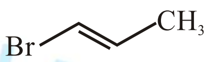
Sol.

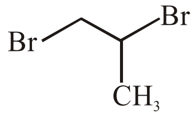


2. 1-methyl ethylene oxide when treated with an excess of HBr produces :

(1) 

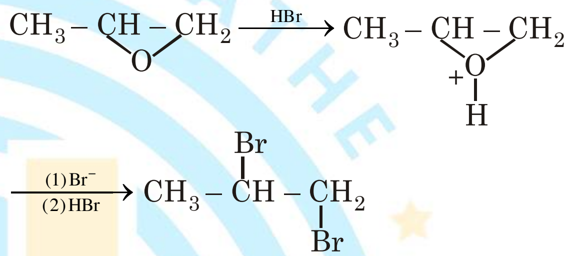
(2) 

(3) 

(4) 

NTA Ans. (4)

Sol.



3. Amongst the following statements, that which was not proposed by Dalton was :

(1) all the atoms of a given element have identical properties including identical mass. Atoms of different elements differ in mass.

(2) chemical reactions involve reorganisation of atoms. These are neither created nor destroyed in a chemical reaction.

(3) when gases combine or reproduced in a chemical reaction they do so in a simple ratio by volume provided all gases are at the same T & P.

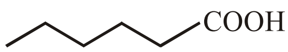
(4) matter consists of indivisible atoms.

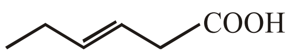
NTA Ans. (3)

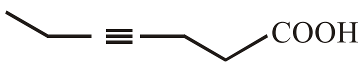
Sol. Option(3) is according to Gaylussac's law of volume combination.

4. What is the product of following reaction ?

$$\text{Hex-3-ynal} \xrightarrow[\substack{\text{(ii) PBr}_3 \\ \text{(iii) Mg/ether} \\ \text{(iv) CO}_2/\text{H}_3\text{O}^+}]{\text{(i) NaBH}_4} ?$$

(1) 

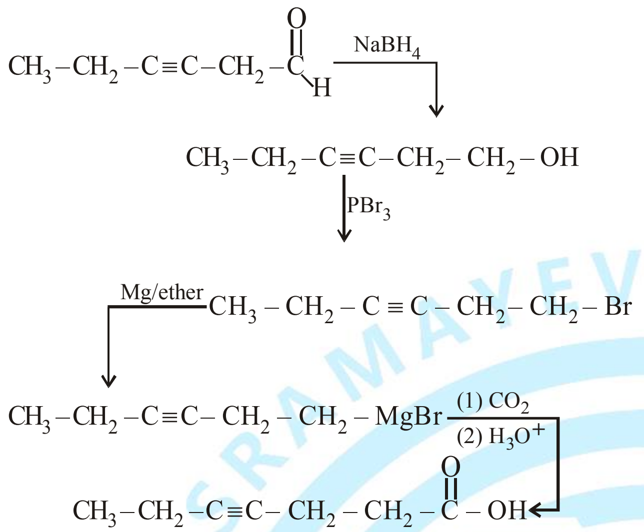
(2) 

(3) 

(4) 

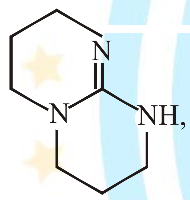
NTA Ans. (3)

Sol.



5. The increasing order of pK_b for the following compounds will be :

$NH_2 - CH = NH,$
(A)



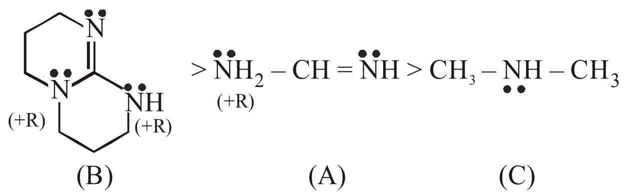
(B)

CH_3NHCH_3
(C)

(1) (A) < (B) < (C) (2) (C) < (A) < (B)
(3) (B) < (A) < (C) (4) (B) < (C) < (A)

NTA Ans. (3)

Sol. Base strength order



increasing order of pk_b order (B < A < C)

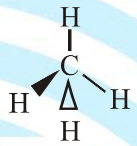
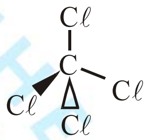
6. The atomic radius of Ag is closest to :
(1) Cu (2) Hg (3) Au (4) Ni

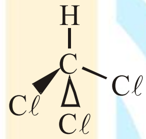
NTA Ans. (3)

Sol. Atomic radius of Ag and Au is nearly same due to lanthanide contraction.

7. The dipole moments of CCl_4, $CHCl_3$ and CH_4 are in the order :
(1) $CH_4 = CCl_4 < CHCl_3$
(2) $CH_4 < CCl_4 < CHCl_3$
(3) $CCl_4 < CH_4 < CHCl_3$
(4) $CHCl_3 < CH_4 = CCl_4$

NTA Ans. (1)

Sol.  $\mu_{net} = 0$  $\mu_{net} = 0$

 $\mu_{net} \neq 0$

8. Given that the standard potentials (E°) of Cu^{2+}/Cu and Cu^+/Cu are 0.34 V and 0.522 V respectively, the E° of Cu^{2+}/Cu^+ is :
(1) +0.158 V (2) 0.182 V
(3) –0.182 V (4) –0.158 V

NTA Ans. (1)

Sol. E^0 ΔG^0

$Cu^{2\oplus} + 2e^{\oplus} \longrightarrow Cu$ 0.34 $= -2F\,(0.34)$

$Cu^{\oplus} + e^{\oplus} \longrightarrow Cu$ 0.522 $= -F\,(0.522)$

$Cu^{2\oplus} + e^{\oplus} \longrightarrow Cu^{+}$

$\Delta G^0 = -2F\,(0.34) - (-F(0.522)) = -F\,(0.68 - 0.522) = -F\,(0.158)$

$$E^0 = \frac{-F(0.158)}{-F} = 0.158V$$

9. In comparison to the zeolite process for the removal of permanent hardness, the synthetic resins method is :

(1) less efficient as it exchanges only anions

(2) more efficient as it can exchange only cations

(3) less efficient as the resins cannot be regenerated

(4) more efficient as it can exchange both cations as well as anions

NTA Ans. (4)

Sol. (a) Zeolite method removes only cations (Ca^{2+} and Mg^{2+} ion) present in hard water

$2NaZ + M^{2+}(aq) \rightarrow MZ_2(s) + 2Na^{+}(aq)$

$(M \rightarrow Mg, Ca)$

(b) Synthetic resin method removes cations (Ca^{2+} and Mg^{2+} ion) and anions (like Cl^-, HCO_3^-, SO_4^{2-} etc.)

(i) $2RNa(s) + M^{2+}(aq) \rightarrow R_2M(s) + 2Na^{+}(aq)$

(Cation exchange resin) $(M \rightarrow Mg, Ca)$

(ii) $RNH_3^{+}\,OH^{-}(s) + X^{-}(aq) \rightarrow RNH_3^{+}\,X^{-}(s) + OH^{-}(aq)$

(Anion exchange resin) ($X^- = Cl^-, HCO_3^-, SO_4^{2-}$ etc)

10. The relative strength of interionic/ intermolecular forces in decreasing order is :

(1) ion-dipole > ion-ion > dipole-dipole

(2) dipole-dipole > ion-dipole > ion-ion

(3) ion-dipole > dipole-dipole > ion-ion

(4) ion-ion > ion-dipole > dipole-dipole

NTA Ans. (4)

Sol. Order is

ion – ion > ion – dipole > dipole – dipole

11. Consider the following reactions :

(a) $(CH_3)_3CCH(OH)CH_3 \xrightarrow{\text{conc.}H_2SO_4}$

(b) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{\text{alc.KOH}}$

(c) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow[\text{It should be }(CH_3)_3CO^{\ominus}K^{\oplus}]{\text{given by NTA }(CH_3)_3O^{\ominus}K^{\oplus}}$

(d) $(CH_3)_2\underset{\displaystyle OH}{\underset{|}{C}}{-}CH_2{-}CHO \xrightarrow{\Delta}$

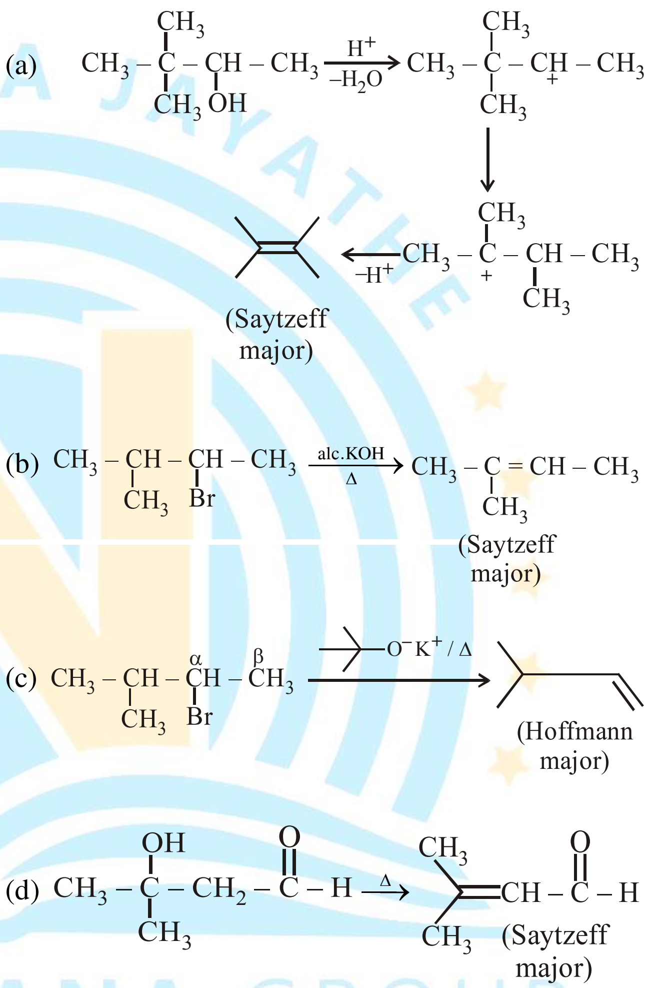
Which of these reaction(s) will not produce Saytzeff product ?

(1) (c) only (2) (a), (c) and (d)

(3) (d) only (4) (b) and (d)

NTA Ans. (1)

Sol.



$(CH_3)_3O^-K^+$ is incorrect representation of potassium tert-butoxide $[(CH_3)_3CO^-K^+]$.

So it is possible that it can be given as **Bonus**

12. The purest form of commercial iron is

(1) scrap iron and pig iron

(2) wrought iron

(3) cast iron

(4) pig iron

NTA Ans. (2)

Sol. Wrought iron is purest from of commercial iron.

13. At 35°C, the vapour pressure of CS_2 is 512 mm Hg and that of acetone is 344 mm Hg. A solution of CS_2 in acetone has a total vapour pressure of 600 mm Hg. The false statement amongst the following is :

(1) heat must be absorbed in order to produce the solution at 35°C

(2) Raoult's law is not obeyed by this system

(3) a mixture of 100 mL CS_2 and 100 mL acetone has a volume < 200 mL

(4) CS_2 and acetone are less attracted to each other than to themselves

NTA Ans. (3)

Sol. The vapour pressure of mixture (= 600 mm Hg) is greater than the individual vapour pressure of its constituents (Vapour pressure of CS_2 = 512 mm Hg, acetone = 344 mm Hg). Hence, the solution formed shows positive deviation from Raoult's law.

$\Rightarrow (1) \Delta_{Sol}H > 0$, (2) Raoult's law is not obeyed

(3) $\Delta_{sol.}$ Volume > 0

(4) CS_2 and Acetone are less attracted to each ether than to themselves.

14. The electron gain enthalpy (in kJ/mol) of fluorine, chlorine, bromine and iodine, respectively are :

(1) – 333, – 349, – 325 and – 296

(2) –296, – 325, – 333 and – 349

(3) – 333, – 325, – 349 and – 296

(4) –349, – 333, – 325 and – 296

NTA Ans. (1)

Sol. Order of electron gain enthalpy (magnitude) is Cl > F > Br > I

15. The number of orbitals associated with quantum numbers n = 5, $m_s = +\frac{1}{2}$ is :

(1) 11 (2) 25 (3) 15 (4) 50

NTA Ans. (2)

Sol. No. of orbitals = $n^2 = 5^2 = 25$

For n = 5, no. of orbitals = n^2 = 25

Total number of orbitals is equal to no. of electrons having $m_s = \frac{1}{2}$

16. Match the following :

(i) Riboflavin	(a) Beriberi
(ii) Thiamine	(b) Scurvy
(iii) Pyridoxine	(c) Cheilosis
(iv) Ascorbic acid	(d) Convulsions

(1) (i)-(c), (ii)-(a), (iii)-(d), (iv)-(b)

(2) (i)-(c), (ii)-(d), (iii)-(a), (iv)-(b)

(3) (i)-(d), (ii)-(b), (iii)-(a), (iv)-(c)

(4) (i)-(a), (ii)-(d), (iii)-(c), (iv)-(b)

NTA Ans. (1)

Sol. (i) Riboflavin $\longrightarrow$ (c) Cheilosis

(ii) Thiamine $\longrightarrow$ (a) Beriberi

(iii) Pyridoxine $\longrightarrow$ (d) Convulsions

(iv) Ascorbic acid $\longrightarrow$ (b) Scurvy

17. The theory that can completely/properly explain the nature of bonding in $[Ni(CO)_4]$ is :

(1) Werner's theory

(2) Crystal field theory

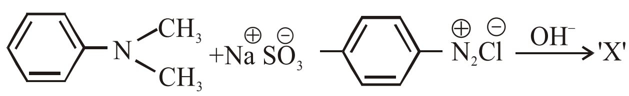
(3) Valence bond theory

(4) Molecular orbital theory

NTA Ans. (4)

Sol. In complex $[Ni(CO)_4]$ decrease in Ni–C bond length and increase in C–O bond length as well as it's magnetic property is explained by MOT.

18. Consider the following reaction :



The product 'X' is used :

(1) in acid base titration as an indicator

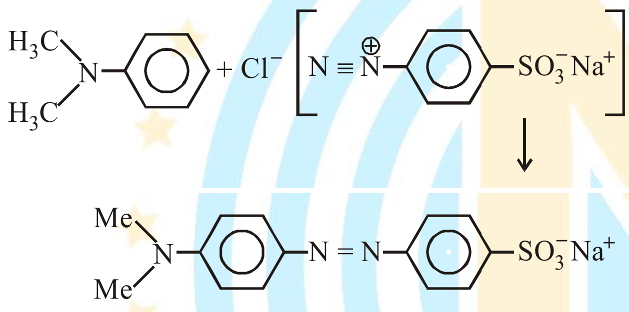
(2) in protein estimation as an alternative to ninhydrin

(3) in laboratory test for phenols

(4) as food grade colourant

NTA Ans. (1)

Sol.



(Methyl orange)

It is an acid base indicator

19. The IUPAC name of the complex $[Pt(NH_3)_2Cl(NH_2CH_3)]Cl$ is :

(1) Diammine (methanamine) chlorido platinum (II) chloride

(2) Bisammine (methanamine) chlorido platinum (II) chloride

(3) Diamminechlorido (aminomethane) platinum(II) chloride

(4) Diamminechlorido (methanamine) platinum (II) chloride

NTA Ans. (4)

20. Oxidation number of potassium in K_2O, K_2O_2 and KO_2 , respectively, is :

(1) +1, +4 and +2

(2) +1, +2 and +4

(3) +1, +1 and +1

(4) +2 , +1 and $+\frac{1}{2}$

NTA Ans. (3)

Sol. Potasium has an oxidation of +1 (only) in combined state.

21. For the reaction ;

$A(l) \rightarrow 2B(g)$

$\Delta U = 2.1$ kcal , $\Delta S = 20$ cal K^{-1} at 300 K

Hence ΔG in kcal is_________ .

NTA Ans. (–2.70)

Sol. $A(\ell) \longrightarrow 2B(g)$

$\Delta U = 2.1$ Kcal , $\Delta S = 20$ cal K^{-1} at 300 K

$\Delta H = \Delta U + \Delta n_g RT$

$$\Delta G = \Delta H - T\Delta S$$

$$\Delta G = \Delta U + \Delta n_g RT - T\Delta S$$

$$= 2.1 + \frac{2 \times 2 \times 300}{1000} - \frac{300 \times 20}{1000}$$

(R = 2 cal K^{-1} mol^{-1})

= 2.1 + 1.2 – 6 = – 2.70 Kcal/mol

22. During the nuclear explosion, one of the products is ^{90}Sr with half life of 6.93 years. if 1 μg of ^{90}Sr was absorbed in the bones of a newly born baby in place of Ca, how much time, in years , is required to reduce it by 90% if it is not lost metabolically______ .

NTA Ans. (23 to 23.03)

Sol. All nuclear decays follow first order kinetics

$$t = \frac{1}{k}\ell n\frac{[A_0]}{[A]}$$

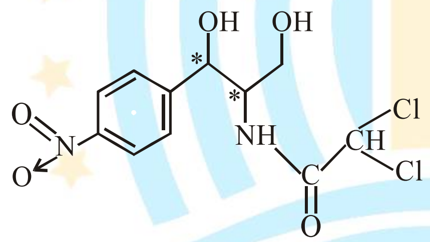
$$= \frac{(t_{1/2})}{0.693}\times 2.303 \ \log_{10}10 \quad = 10 \times 2.303 \times 1$$

= 23.03 years

23. The number of chiral carbons in chloramphenicol is _______ .

NTA Ans. (2)

Sol.



Chloramphenicol

24. Two solutions A and B, each of 100 L was made by dissolving 4g of NaOH and 9.8 g of H_2SO_4 in water, respectively. The pH of the resultant solution obtained from mixing 40 L of solution A and 10 L of solution B is_______.

NTA Ans. (10.60 to 10.60)

Sol. 4 gm of NaOH in 100 L sol. ⇒ 10^{-3} M sol.

9.8 gm of H_2SO_4 in 100 L sol. ⇒ 10^{-3} M sol.

Mixture : 40L of 10^{-3} M NaOH and 10 L of 10^{-3} M H_2SO_4 sol.

Final Conc. of OH^-

$$= \frac{10^{-3}(40\times 1 - 10\times 1\times 2)}{40+10} = 6\times 10^{-4} M$$

pOH = – log (6 ×10^{-4})

= 4 – log 6 = 4 – 0.60 = 3.40

pH = 14 – 3.40 = 10.60

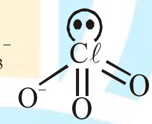
25. Chlorine reacts with hot and concentrated NaOH and produces compounds (X) and (Y). Compound (X) gives white precipitate with silver nitrate solution. The average bond order between Cl and O atoms in (Y) is ______.

NTA Ans. (1.66 to 1.67)

Sol. $3Cl_2 + 6NaOH \rightarrow \underset{(X)}{5NaCl} + \underset{(X)}{NaClO_3} + 3H_2O$

$\underset{(X)}{NaCl} + AgNO_3 \rightarrow AgCl \downarrow + NaNO_3$

ClO_3^-



Bond order of Cl–O Bond = $1 + \frac{2}{3} = \frac{5}{3}$

= 1.66 or 1.67

FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Tuesday 07th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

MATHEMATICS | TEST PAPER WITH ANSWER & SOLUTION

1. If $g(x) = x^2 + x - 1$ and

$(g\circ f)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to

(1) $\frac{3}{2}$ (2) $-\frac{1}{2}$ (3) $-\frac{3}{2}$ (4) $\frac{1}{2}$

NTA Ans. (2)

Sol. $g(x) = x^2 + x - 1$

$g(f(x)) = 4x^2 - 10x + 5$

$= (2x - 2)^2 + (2 - 2x) - 1$

$= (2 - 2x)^2 + (2 - 2x) - 1$

$\Rightarrow f(x) = 2 - 2x$

$f\left(\frac{5}{4}\right) = \frac{-1}{2}$

2. If $\text{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point (x,y) lies on a :

(1) circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2}\right)$

(2) circle whose diameter is $\frac{\sqrt{5}}{2}$

(3) straight line whose slope is $\frac{3}{2}$

(4) straight line whose slope is $-\frac{2}{3}$

NTA Ans. (2)

Sol. $\text{Re}\left(\frac{z-1}{2z+i}\right) = 1$

Put $z = x + iy$

$$\text{Re}\left(\frac{(x+iy)-1}{2(x+iy)+i}\right) = 1$$

$$\text{Re}\left(\left(\frac{(x-1)+iy}{2x+i(2y+1)}\right)\left(\frac{2x-i(2y+1)}{2x-i(2y+1)}\right)\right) = 1$$

$\Rightarrow 2x^2 + 2y^2 + 2x + 3y + 1 = 0$

$$x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0$$

$\Rightarrow$ locus is a circle whose

Centre is $\left(-\frac{1}{2}, -\frac{3}{4}\right)$ and radius $\frac{\sqrt{5}}{4}$

$\Rightarrow$ diameter $= \frac{\sqrt{5}}{2}$

3. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is :

(1) $\frac{21}{2}$ (2) 27 (3) 16 (4) 7

NTA Ans. (3)

Sol. Let the A.P is

$a - 2d,\ a - d,\ a,\ a + d,\ a + 2d$

$\because$ sum $= 25 \Rightarrow a = 5$

Product = 2520

$(25 - 4d^2)(25 - d^2) = 504$

$4d^4 - 125d^2 + 121 = 0$

$\Rightarrow d^2 = 1, \frac{121}{4}$

$\Rightarrow d = \pm 1, \pm\frac{11}{2}$

$d = \pm 1$ is rejected because none of the term can be $\frac{-1}{2}$.

$\Rightarrow d = \pm\frac{11}{2}$

$\Rightarrow$ AP will be $-6, -\frac{1}{2}, 5, \frac{21}{2}, 16$

Largest term is 16.

4. If

$$y(\alpha)=\sqrt{2\left(\frac{\tan\alpha+\cot\alpha}{1+\tan^2\alpha}\right)+\frac{1}{\sin^2\alpha}},\alpha\in\left(\frac{3\pi}{4},\pi\right),$$

then $\frac{dy}{d\alpha}$ at $\alpha=\frac{5\pi}{6}$ is :

(1) 4 (2) $-\frac{1}{4}$ (3) $\frac{4}{3}$ (4) –4

NTA Ans. (1)

Sol. $y(\alpha)=\sqrt{2\frac{(\tan\alpha+\cot\alpha)}{1+\tan^2\alpha}+\frac{1}{\sin^2\alpha}}$, $\alpha\in\left(\frac{3\pi}{4},\pi\right)$

$=\frac{|\sin\alpha+\cos\alpha|}{|\sin\alpha|}=\frac{-(\sin\alpha+\cos\alpha)}{\sin\alpha}$

$= -1 - \cot\alpha$

$y'(\alpha) = \text{cosec}^2\alpha$

$y'\left(\frac{5\pi}{6}\right)=4$

5. Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1&1&1\\1&\alpha&\alpha^2\\1&\alpha^2&\alpha^4\end{bmatrix}$, then the matrix A^{31} is equal to:

(1) A^3 (2) A (3) A^2 (4) I_3

NTA Ans. (1)

Sol. $x^2 + x + 1 = 0$

$\alpha = \omega$

$\alpha^2 = \omega^2$

$$A=\frac{1}{\sqrt{3}}\begin{bmatrix}1&1&1\\1&\omega&\omega^2\\1&\omega^2&\omega\end{bmatrix}$$

$$A^2=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$$

$\Rightarrow A^4 = A^2 . A^2 = I_3$

$A^{31} = A^{28}. A^3 = A^3.$

6. If $y = mx + 4$ is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to :

(1) 128 (2) –64 (3) –128 (4) –32

NTA Ans. (3)

Sol. $y = mx + 4$ is tangent to $y^2 = 4x$

$\Rightarrow m = \frac{1}{4}$

$y=\frac{1}{4}x+4$ is tangent to $x^2 = 2by$

$\Rightarrow x^2-\frac{b}{2}x-8b=0$

$\Rightarrow D = 0$

$b^2 + 128b = 0$

$\Rightarrow b = -128, 0$

$b \neq 0 \Rightarrow b = -128$

7. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is :

(1) $\sqrt{3}$ (2) $2\sqrt{3}$ (3) $3\sqrt{2}$ (4) $\frac{3}{\sqrt{2}}$

NTA Ans. (3)

Sol. Given $2ae = 6 \Rightarrow \boxed{ae=3}$ ......(1)

and $\frac{2a}{e}=12\Rightarrow\boxed{a=6e}$ ....(2)

from (1) and (2)

$6e^2 = 3 \Rightarrow \boxed{e=\frac{1}{\sqrt{2}}}$

$\Rightarrow \boxed{a=3\sqrt{2}}$

Now, $b^2 = a^2 (1 - e^2)$

$\Rightarrow b^2 = 18\left(1-\frac{1}{2}\right)=9$

Length of L.R $=\frac{2(9)}{3\sqrt{2}}=3\sqrt{2}$

8. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for k = 3, 4, 5 otherwise X takes the value –1. Then the expected value of X, is :

(1) $\frac{3}{16}$ (2) $-\frac{3}{16}$ (3) $\frac{1}{8}$ (4) $-\frac{1}{8}$

NTA Ans. (3)

Sol.

k	0	1	2	3	4	5
P(k)	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

Expected value $= \sum XP(k)$

$$-\frac{1}{32}-\frac{12}{32}-\frac{11}{32}+\frac{15}{32}+\frac{8}{32}+\frac{5}{32}$$

$$=\frac{28-24}{32}=\frac{4}{32}=\frac{1}{8}$$

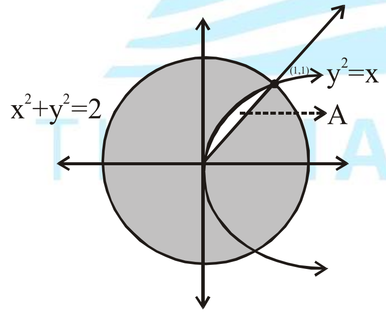
9. The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is :

(1) $\frac{1}{3}(12\pi-1)$ (2) $\frac{1}{6}(12\pi-1)$

(3) $\frac{1}{6}(24\pi-1)$ (4) $\frac{1}{3}(6\pi-1)$

NTA Ans. (2)

Sol.



$$A=\int_0^1\left(\sqrt{x}-x\right)dx$$

$$=\left[\frac{2}{3}x^{3/2}-\frac{x^2}{2}\right]_0^1=\frac{1}{6}$$

Required Area : $\pi r^2-\frac{1}{6}=\frac{1}{6}(12\pi-1)$

10. Let $x^k + y^k = a^k$, (a, K > 0) and $\frac{dy}{dx}+\left(\frac{y}{x}\right)^{\frac{1}{3}}=0$, then k is :

(1) $\frac{3}{2}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{4}{3}$

NTA Ans. (3)

Sol. $x^k + y^k = a^k$ (a ,k > 0)

$$kx^{k-1}+ky^{k-1}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}+\left(\frac{x}{y}\right)^{k-1}=0 \Rightarrow k-1=-\frac{1}{3} \Rightarrow k = 2/3$$

11. If y = y(x) is the solution of the differential equation, $e^y\left(\frac{dy}{dx}-1\right)=e^x$ such that $y(0)=0$, then y(1) is equal to :

(1) $2 + \log_e 2$ (2) 2e

(3) $\log_e 2$ (4) $1 + \log_e 2$

NTA Ans. (4)

Sol. $e^y\frac{dy}{dx}-e^y=e^x$, Let $e^y = t$

$$\Rightarrow e^y\frac{dy}{dx}=\frac{dt}{dx}$$

$$\frac{dt}{dx}-t=e^x$$

$$\text{I.F.} = e^{\int -dx}=e^{-x}$$

$t e^{-x} = x + c \Rightarrow e^{y-x} = x + c$

$y(0) = 0 \Rightarrow c = 1$

$e^{y-x} = x + 1 \Rightarrow y(1) = 1 + \log_e 2$

12. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is :

(1) $\frac{5}{2}(6!)$ (2) 5^6 (3) $\frac{1}{2}(6!)$ (4) 6!

NTA Ans. (1)

Sol. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 is

$$^5C_1 \times \frac{6!}{2!}$$

13. Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is :

(1) (6, 5, –2) (2) (4, 3, 2)

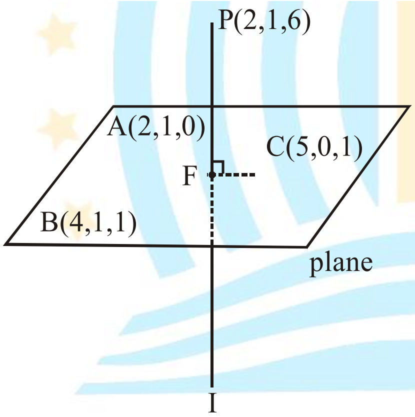
(3) (3, 4, –2) (4) (6, 5, 2)

NTA Ans. (1)

Sol. Plane passing through : (2, 1, 0), (4, 1, 1) and (5, 0, 1)

$$\begin{vmatrix} x-2 & y-1 & z \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ x + y – 2z = 3



Let I and F are respectively image and foot of perpendicular of point P in the plane.

eq^n of line PI $\frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = \lambda(\text{say})$

Let I (λ + 2 , λ + 1, –2λ + 6)

$$\Rightarrow F\left(2+\frac{\lambda}{2}, 1+\frac{\lambda}{2}, -\lambda+6\right)$$

F lies in the plane

$$\Rightarrow 2+\frac{\lambda}{2}+1+\frac{\lambda}{2}+2\lambda-12-3=0$$

$\Rightarrow \lambda = 4$

$\Rightarrow$ I (6, 5, –2)

14. A vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k} (\alpha, \beta \in R)$ lies in the plane of the vectros $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then:

(1) $\vec{a}.\hat{i} + 1 = 0$ (2) $\vec{a}.\hat{i} + 3 = 0$

(3) $\vec{a}.\hat{k} + 4 = 0$ (4) $\vec{a}.\hat{k} + 2 = 0$

NTA Ans. (4)

Sol. $\vec{a} = \lambda\left(\hat{b} + \hat{c}\right) = \lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}} + \frac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$

$$\vec{a} = \frac{\lambda}{3\sqrt{2}}\left(4\hat{i} + 2\hat{j} + 4\hat{k}\right) \Rightarrow \frac{\lambda}{3\sqrt{2}}(4\hat{i} + 2\hat{j} + 4\hat{k})$$

$$= \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$$

$\Rightarrow \alpha = 4$ and $\beta = 4$

So, $\vec{a} = 4\hat{i} + 2\hat{j} + 4\hat{k}$

None of the given options is correct

15. If f (a + b + 1 – x) = f(x), for all x, where a and b are fixed positive real numbers, then $\frac{1}{a+b}\int_a^b x(f(x) + f(x+1))dx$ is equal to :

(1) $\int_{a+1}^{b+1} f(x)dx$ (2) $\int_{a+1}^{b+1} f(x+1)dx$

(3) $\int_{a-1}^{b-1} f(x+1)dx$ (4) $\int_{a-1}^{b-1} f(x)dx$

NTA Ans. (1)

Sol. f(x + 1) = f (a + b – x)

$$I=\frac{1}{(a+b)}\int_a^b x\big(f(x)+f(x+1)\big)dx \quad .....(1)$$

$$I=\frac{1}{(a+b)}\int_a^b (a+b-x)\big(f(x+1)+f(x)\big)\,dx \quad ...(2)$$

from (1) and (2)

$$2I=\int_a^b \big(f(x)+f(x+1)\big)\,dx$$

$$2I=\int_a^b f\big(a+b-x\big)dx+\int_a^b f(x+1)dx$$

$$2I=2\int_a^b f(a+1)dx \Rightarrow I=\int_a^b f(x+1)dx$$

$$=\int_{a-1}^{b-1} f(x)dx$$

OR

$$I=\frac{1}{(a+b)}\int_a^b x\big(f(x)+f(x+1)\big)dx \quad ...(1)$$

$$=\frac{1}{(a+b)}\int_a^b (a+b-x)\big(f(a+b-x)+f(a+b+1-x)\big)dx$$

$$I=\frac{1}{(a+b)}\int_a^b (a+b-x)\big(f(x+1)+f(x)\big)dx \quad ...(2)$$

equation (1) + (2)

$$2I=\frac{1}{(a+b)}\int_a^b (a+b)\big(f(x+1)+f(x)\big)dx$$

$$I=\frac{1}{2}\left[\int_a^b f(x+1)dx+\int_a^b f(x)dx\right]$$

$$=\frac{1}{2}\left[\int_a^b f(a+b+1-x)dx+\int_a^b f(x)dx\right]$$

$$=\frac{1}{2}\left[\int_a^b f(x)dx+\int_a^b f(x)dx\right]$$

$$I=\int_a^b f(x)dx$$

Let x = T + 1

$$=\int_{a-1}^{b} f(T+1)dT$$

$$I=\int_{a-1}^{b-1} f(x+1)dx$$

16. Let the function, $f : [-7, 0] \to R$ be continuous on [–7, 0] and differentiable on (–7, 0). If $f(-7) = -3$ and $f'(x) \le 2$, for all $x \in (-7, 0)$, then for all such functions f, $f(-1) + f(0)$ lies in the interval :

(1) [–6, 20] (2) (–∞, 20]

(3) (– ∞, 11] (4) [–3, 11]

NTA Ans. (2)

Sol. Using LMVT in [–7, –1]

$$\frac{f(-1)-f(-7)}{-1-(-7)}\le 2$$

$f(-1) - f(-7) \le 12$

$\Rightarrow f(-1) \le 9$ ....(1)

Using LMVT in [–7, 0]

$$\frac{f(0)-f(-7)}{0-(-7)}\le 2$$

$f(0) - f(-7) \le 14$

$f(0) \le 11$ ....(2)

from (1) and (2)

$f(0) + f(-1) \le 20$

17. If the system of linear equations

2x + 2ay + az = 0

2x + 3by + bz = 0

2x + 4cy + cz = 0,

where a, b, c ∈ R are non-zero and distinct; has a non-zero solution, then :

(1) a, b, c are in A.P.

(2) a + b + c = 0

(3) a, b, c are in G.P.

(4) $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P.

NTA Ans. (4)

Sol. For non-zero solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0, \Rightarrow \begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

$\Rightarrow$ (3b – 2a) (c –a) – (b – a) (4c – 2a) = 0

$\Rightarrow$ 2ac = bc + ab

$$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

Hence $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

18. Let α and β be two real roots of the equation $(k + 1)\tan^2 x - \sqrt{2} \cdot \lambda \tan x = (1 - k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is ;

(1) 5 (2) 10 (3) $5\sqrt{2}$ (4) $10\sqrt{2}$

NTA Ans. (2)

Sol. $\tan\alpha + \tan\beta = \frac{\lambda\sqrt{2}}{k+1}$

$\tan\alpha . \tan\beta = \frac{k-1}{k+1}$

$$\tan(\alpha + \beta) = \frac{\frac{\lambda\sqrt{2}}{k+1}}{1 - \frac{k-1}{k+1}} = \frac{\lambda\sqrt{2}}{2} = \frac{\lambda}{\sqrt{2}}$$

$$\Rightarrow \frac{\lambda^2}{2} = 50 \Rightarrow \lambda = 10 \,\&\, -10$$

19. The logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :

(1) p (2) q (3) ~p (4) ~q

NTA Ans. (3)

Sol. $(p \to q) \wedge (q \to \sim p)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee \sim p)$

$\equiv \sim p \vee (q \wedge \sim q)$

$\equiv \sim p \vee C \equiv \sim p$

20. The greatest positive integer k, fr which $49^k + 1$ is a factor of the sum

$49^{125} + 49^{124} + \ldots\ 49^2 + 49 + 1$, is :

(1) 32 (2) 60 (3) 63 (4) 65

NTA Ans. (3)

Sol. $1 + 49 + 49^2 + \ldots + 49^{12}$

$$= (49)^{126} - 1 = (49^{63} + 1)\frac{(49^{63} - 1)}{(48)}$$

So greatest value of k = 63

21. $\lim\limits_{x \to 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$ is equal to ____________.

NTA Ans. (36)

Sol. $$\lim_{x \to 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}} \Rightarrow \lim_{x \to 2} \frac{3^{2x} - 12.3^x + 27}{3^{x/2} - 3}$$

$$= \lim_{x \to 2} \frac{(3^x - 9)(3^x - 3)}{(3^{x/2} - 3)}$$

$$= \lim_{x \to 2} \frac{(3^{x/2} + 3)(3^{x/2} - 3)(3^x - 3)}{(3^{x/2} - 3)}$$

= 36

22. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then m + n is equal to ________.

NTA Ans. (18)

Sol. Variance of first 'n' natural numbers = $\frac{n^2-1}{12}=10$

$\Rightarrow$ n = 11

and variance of first 'm' even natural numbers

= $4\left(\frac{m^2-1}{12}\right)\Rightarrow\frac{m^2-1}{3}=16\Rightarrow m=7$

m + n = 18

23. If the sum of the coefficients of all even powers of x in the product

$(1 + x + x^2 + ... + x^{2n})(1 - x + x^2 - x^3 + ... + x^{2n})$

is 61, then n is equal to _________.

NTA Ans. (30)

Sol. Let $(1 + x + x^2 + \ldots + x^{2n})(1 - x + x^2 - x^3 + \ldots + x^{2n})$

$= a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \ldots + a_{4n}x^{4n}$

So,

$a_0 + a_1 + a_2 + \ldots + a_{4n} = 2n + 1$ ...(1)

$a_0 - a_1 + a_2 - a_3 \ldots + a_{4n} = 2n + 1$ ...(2)

$\Rightarrow a_0 + a_2 + a_4 + \ldots + a_{4n} = 2n + 1$

$\Rightarrow 2n + 1 = 61$ $\Rightarrow n = 30$

24. Let A(1, 0), B(6, 2) and $C\left(\frac{3}{2},6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6},-\frac{1}{3}\right)$, is __________.

NTA Ans. (5)

Sol. P is centroid of the triangle ABC

$\Rightarrow P \equiv \left(\frac{17}{6},\frac{8}{3}\right)$

$\Rightarrow$ PQ = 5

25. Let S be the set of points where the function, $f(x) = |2-|x-3||$, x∈R, is not differentiable.

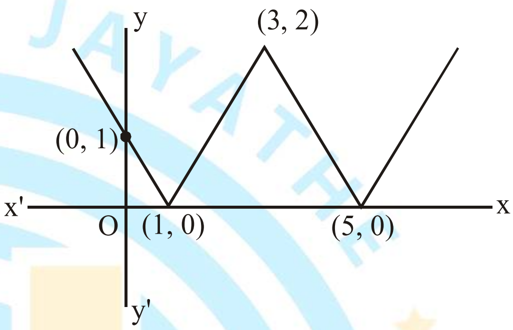
Then $\sum_{x\in S} f(f(x))$ is equal to __________.

NTA Ans. (3)

Sol. $f(x) = |2 - |x - 3||$

f is not differentiable at

x = 1, 3, 5



$\Rightarrow \sum_{x\in S} f(f(x)) = f(f(1)) + f(f(3)) + f(f(5))$

= f(0) + f(2) + f(0)

= 1 + 1 + 1 = 3